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On Discrimination with Competition between Groups*

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Abstract

Statistical discrimination explains that two ex ante identical groups can have two different qualifications due to asymmetric information and self-fulfilling equilibria. In the typical statistical discrimination models, however, there is no interaction between groups. This paper offers a statistical discrimination model with a continuous signaling in which two groups compete for employment. We compare exclusive equilibria, in which no worker in one group makes a human capital investment, with symmetric equilibria, and show that discrimination as well as non-discrimination can be Pareto optimal under a certain environment.

Keywords Statistical Discrimination, Group Inequality, Asymmetric Information

JEL Classification D63, D82, J71

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Theories of statistical discrimination (Arrow (1973), Phelps (1972) and Coate & Loury (1993)) explain that two *ex ante* identical groups can have two different qualification levels as a result of asymmetric information and *self-fulfilling equilibria*.

In typical statistical discrimination models, there is no interaction between the two groups. Hence, the changes in one group's qualification level do not affect the other's qualification level, and both groups can have a higher qualification level, at least in theory. It follows that discrimination is always not Pareto optimal. However, in the real world, different groups compete for employment or admission in a fixed number of positions, and most conflicts between them occur under this type of environment.[1]

This paper offers a statistical discrimination model with a *continuous* signaling in which two groups compete for employment. We deliberately endow an environment with many employers and workers in which opportunities are limited. Each employer is randomly matched with two workers from the entire worker population, which consists of two *ex ante* identical sub-groups, and selects *at most* one of them. In particular, each employer chooses a worker from group i when (i) the worker's signal is stronger than a test standard, and (ii) group i's probability of being qualified is greater than the other's. In the typical statistical discrimination models, only the former condition (i) is imposed.

We assume a general *continuous* signaling which is widely used in the literature following Coate & Loury (1993). [2] The general signaling structure, however, limits the scope of the analysis: finding the full characterization of interior asymmetric equilibria is not tractable. The purpose of this paper is to compare *exclusive equilibria,* in which only workers from one group are selected for a certain type of job, with symmetric equilibria, in which both groups with the same qualification level apply for it.[3] The primary question this paper asks is what types of exclusive and symmetric equilibria are feasible, and what the related welfare implications are in that case. Consider, for example, a situation in which one group has been historically totally excluded from a certain position, and in which a policy is introduced to achieve equalization, a symmetric equilibrium, between the two groups. In the typical statistical discrimination models, every qualification level of the advantaged group in an exclusive equilibrium can be *replicated* for the disadvantaged in a symmetric equilibrium, which is a Pareto improvement.

[1]The most notable example is the lawsuit against the law school of the University of Michigan (*New York Times*, May 11, 1999)

[2]A continuous signaling in this context means that a worker can emit continuous test results.

[3]Exclusive equilibria can be called boundary asymmetric equilibria.

This paper shows that when there are multiple exclusive equilibria with two qualification levels of the advantaged group, there can be two types of symmetric equilibria: one symmetric qualification level is lower than a low exclusive qualification level of the advantaged group, and the other symmetric qualification level is higher than a high exclusive qualification level. Even in the case from an exclusive equilibrium to the high symmetric equilibrium, every worker type in the advantaged group is worse off if the negative effect from greater competition with a *more qualified* disadvantaged group is large enough to outweigh the positive effect from the decrease in its own's test standard given a higher qualification level. Therefore, discrimination as well as non-discrimination can be Pareto optimal.

Mailath et al. (2000) and Moro & Norman (2004) offer models with an interaction between different groups through *externality.* In the former, with a search approach, one group's search benefit depends on the other group's qualification level, and in the latter, with a general equilibrium model, one group's marginal product depends on the other group's. Neither study addresses strategic interplay between workers from different groups under direct competition.[4] Mialon & Yoo (2017) introduce competition between groups with a *discrete* signaling, providing a complete characterization of interior symmetric and asymmetric equilibria, and show that employers benefit from discrimination against a minority group.[5]

We introduce the model in Section 1 and provide the main results in Section 2. Concluding remarks can be found in Section 3.

1. MODEL

Consider a market in which there are many identical employers and workers. Workers belong to one of two distinct groups, A and B, and membership is publicly observable with zero cost. Each worker from group $i \in \{A,B\}$ decides whether to make a human capital investment to become qualified or not, denoted by $q_i = 1$ or 0, and in contrast to group identity, each worker's qualifications are known only to him- or herself. Each worker's investment cost c_i is drawn from a continuous distribution. We assume a *symmetric* environment for the two groups such that half the total population is from group A, and they have the an identical cost CDF F which has a support $[\underline{c}, \bar{c}]$ with $0 \leq \underline{c} < \bar{c}$.[6] When a worker

[4] Lang et al. (2005) feature multiple job applicants, but no "competing procedure" for the hiring; if the employer receives more than one application, he chooses to hire one applicant at random.

[5] See also Yoo (2013) for an extended analysis with a repeated game.

[6] The results in the paper are robust to different population shares.

is matched with an employer, the worker's test result emits, and the test result θ_i is drawn from a continuously differentiable CDF $G_{q_i}(\theta_i)$ for $q_i = 1, 0$ with its support $[\underline{\theta}, \overline{\theta}]$ and $\underline{\theta} < \overline{\theta}$. Let its density $g_{q_i}(\theta_i) > 0$ for all θ, q_i and define $\phi : [\underline{\theta}, \overline{\theta}] \to \mathbb{R}_{++}$ by $\phi(\theta_i) \equiv g_0(\theta_i)/g_1(\theta_i)$. We assume that[7]

$$\phi(\theta_i) \text{ is strictly decreasing,} \tag{1}$$

which implies that the likelihood that a higher value of the test result emits with $q_i = 1$ is greater.

Each employer is randomly matched with two workers from the whole population, and after observing their test results, the employer decides to select at most one worker for a position. Each employer gains a return $x > 0$ if a worker is qualified, 0 otherwise, and pays a reward $v \in (0, x)$, which is fixed as in Coate & Loury (1993) and Blume (2005), for a selected worker.[8] Hence, the selected worker obtains the gross benefit v, and the non-selected worker obtains the normalized gross benefit 0.

Let $\Theta \equiv [\underline{\theta}, \overline{\theta}]^2$ and $\theta \in \Theta$. Each worker's strategy is a mapping $Q_i : [\underline{c}, \overline{c}] \to \{0, 1\}$, each employer's strategy is a mapping $E : \Theta \to \{i, j, \phi\}$. Each group i worker's payoff, when selected, is

$$u_i \equiv v - c_i q_i.$$

Each employer's payoff from hiring a worker from group i is

$$u_F \equiv x q_i - v.$$

Since each worker's *type* c_i is not included in the benefit part, and his decision is binary, each worker's optimal strategy is a "cutoff strategy." That is, there exists $k \in [\underline{c}, \overline{c}]$ such that a worker becomes qualified if $c_i < k$ but unqualified if $c_i > k$.

We denote by $\mu : [\underline{\theta}, \overline{\theta}] \times [\underline{c}, \overline{c}] \to [0, 1]$ each employer's posterior probability that a worker from group i is qualified given signal θ_i and the employer's belief about group i's cutoff k_i. By Bayes' rule, for $k_i \in (\underline{c}, \overline{c}]$,

$$\mu(\theta_i, k_i) \equiv \frac{g_1(\theta_i) F(k_i)}{g_1(\theta_i) F(k_i) + g_0(\theta_i)(1 - F(k_i))},$$

which can be succinctly rewritten as

$$\mu(\theta_i, k_i) \equiv \begin{cases} 1/(1 + \phi(\theta_i)\pi(k_i)) & \text{if} \quad k_i \in (\underline{c}, \overline{c}], \\ 0 & \text{if} \quad k_i = \underline{c}, \end{cases} \tag{2}$$

[7] Note that this implies that $G_0(\theta_i) > G_1(\theta_i)$ for all θ_i.

[8] According to Petersen & Saporta (2004), within-job *wage* discrimination is least prevalent and least important since it is illegal and easy to document.

where $\pi : (\underline{c}, \overline{c}] \to \mathbb{R}_+$ is defined by

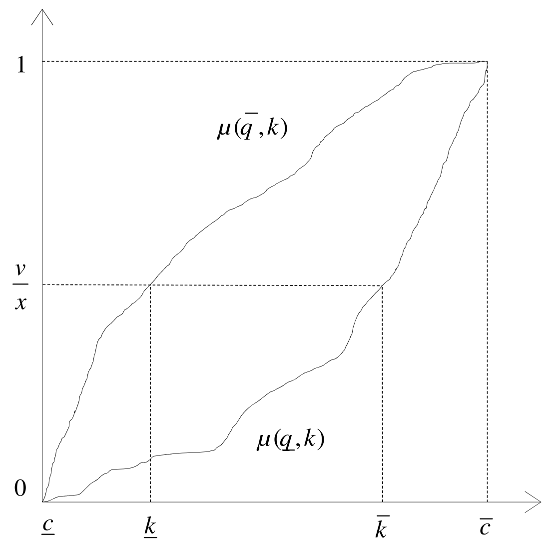
$$\pi(k_i) \equiv \frac{1 - F(k_i)}{F(k_i)}.$$

The employer's expected payoff is $\mu(\theta_i, k_i)x - v$, so the employer's sequentially rational strategy is to select a worker only when $\mu(\theta_i, k_i) \geq v/x$. Define a *test standard* $s(k_i)$ such that it is optimal for the employer to select workers of group i if and only if a signal exceeds the standard $s(k_i)$. Then, there exist $\underline{k}, \overline{k} \in (\underline{c}, \overline{c})$ and a unique function $\widehat{\theta}(k_i)$ such that

$$s(k_i) = \begin{cases} \underline{\theta} & \text{if} \quad k_i > \overline{k}, \\ \widehat{\theta}(k_i) & \text{if} \quad k_i \in [\underline{k}, \overline{k}], \\ \overline{\theta} & \text{if} \quad k_i < \underline{k}, \end{cases} \tag{3}$$

where $\mu(\underline{\theta}, \overline{k}) = v/x$, $\mu(\overline{\theta}, \underline{k}) = v/x$ and $\mu(\widehat{\theta}(k_i), k_i) = v/x$. It can be shown that s is a decreasing function of k_i. Figure 1 visualizes $\underline{k}$ and $\overline{k}$.

Figure 1: $\underline{k}$ and $\overline{k}$



Suppose that the employer is matched with one from group i and another from group j. Given $\theta \in \Theta$, a worker from group i will be selected if

(i) the worker's signal is greater than the standard, $\theta_i > s(k_i)$, and
(ii) group i's probability of being qualified is greater than the other's, $\mu(\theta_i, k_i) > \mu(\theta_j, k_j)$.

With an additional notation $\Theta(k_i) \equiv [s(k_i), \overline{\theta}] \times [\underline{\theta}, \overline{\theta}]$, the probability $P : \{0,1\} \times [\underline{c}, \overline{c}]^2 \to [0,1]$ that a member with qualification q_i from group i is selected can be derived as follows:

$$P(q_i, k_i, k_j) \equiv F(k_j) \int_{\Theta(k_i)} \zeta(\theta, k_i, k_j)\, dG_{q_i}(\theta_i) \times G_1(\theta_j) + (1 - F(k_j)) \int_{\Theta(k_i)} \zeta(\theta, k_i, k_j)\, dG_{q_i}(\theta_i) \times G_0(\theta_j), \tag{4}$$

where $\zeta : \Theta \times [\underline{c}, \overline{c}]^2 \to \{0,1\}$ is

$$\zeta(\theta, k_i, k_j) \equiv \begin{cases} 1 & \text{if} \quad \mu(\theta_i, k_i) > \mu(\theta_j, k_j), \\ 1/2 & \text{if} \quad \mu(\theta_i, k_i) = \mu(\theta_j, k_j), \\ 0 & \text{if} \quad \mu(\theta_i, k_i) < \mu(\theta_j, k_j). \end{cases}$$

The probability P includes not only the employer's beliefs about group i's qualifications but also group i's beliefs about group j's qualifications. Hence, if a worker from group i becomes qualified, the increase in the probability given the belief (k_i, k_j) can be written as the function $\beta : [\underline{c}, \overline{c}]^2 \to [0,1]$:

$$\beta(k_i, k_j) \equiv P(1, k_i, k_j) - P(0, k_i, k_j).$$

As a function of k_i and k_j, β has the following properties.

Lemma 1. *(i) For each $k_i \in [\underline{c}, \underline{k}]$, $\beta(k_i, k_j) = 0$ for all $k_j \in [\underline{c}, \overline{c}]$.*

(ii) $\beta(\overline{c}, k_j) = 0$ for all $k_j \in [\underline{c}, \overline{c}]$.

(iii) $\beta(k_i, k_j)$ is a continuous function of k_i on $[\underline{c}, \overline{c}]$ given a fixed $k_j \in [\underline{c}, \overline{c})$.

(iv) $\beta(k, k)$ is a continuous function of k on $[\underline{c}, \overline{c})$.

Proof. See Appendix. □

Since each employer is randomly matched with two workers from the whole population, every worker in a group has $1/2$ chance to compete with a worker from the same group, and $1/2$ chance to compete with a worker from the other group. Hence, we define $\Phi(k_i, k_j)\, v$ as *the incentive to become qualified*, where

$$\Phi(k_i, k_j) \equiv \frac{1}{2}\beta(k_i, k_i) + \frac{1}{2}\beta(k_i, k_j).$$

Thus, an equilibrium[9] is defined as a combination $(k_A^*, k_B^*) \in [\underline{c}, \overline{c}]^2$ such that for each $i \in \{A, B\}$,

$$\begin{array}{lll} \Phi(k_i^*, k_j^*)v \leq k_i^* & \text{if} & k_i^* = \underline{c}, \\ \Phi(k_i^*, k_j^*)v = k_i^* & \text{if} & k_i^* \in (\underline{c}, \overline{c}), \\ \Phi(k_i^*, k_j^*)v \geq k_i^* & \text{if} & k_i^* = \overline{c}. \end{array} \tag{5}$$

If $k_A^* = k_B^* = k_s^* \in (\underline{c}, \overline{c})$, we call (k_A^*, k_B^*) a non-trivial symmetric equilibrium, and (5) becomes

$$\Phi(k_s^*, k_s^*)v = \beta\,(k_s^*, k_s^*)\,v = k_s^*.$$

The existence of multiple equilibria does not imply the existence of asymmetric equilibria as in the typical statistical discrimination models. If $k_A^* \neq k_B^*$, we call (k_A^*, k_B^*) an asymmetric equilibrium. For the general continuous signaling structure, the full characterization of asymmetric equilibria is not easy.[10] We focus on an asymmetric equilibrium in which no worker in one group makes a human capital investment such that $(k_A^*, k_B^*) = (k_e^*, \underline{c})$ with $k_e^* > \underline{c}$. We call this an exclusive (asymmetric) equilibrium.

2. MAIN RESULTS

First, we show that there exist non-trivial multiple symmetric equilibria and multiple exclusive equilibria.

Proposition 1. *For x, F and (G_1, G_0), there exists $v > 0$ such that there exist non-trivial multiple symmetric equilibria and multiple exclusive equilibria.*

[9] If an investment cost c is interpreted as a *type*, it is the same as a perfect *Bayesian* equilibrium. Formally, Q_A^*, Q_B^* and E^* with the belief μ is a perfect Bayesian equilibrium if for each $c_i \in [\underline{c}, \bar{c}]$ of every group $i \in \{A, B\}$,

$$Q_i^*(c_i) = \arg\max_{q_i \in \{0,1\}} u_i(q_i, E^*, Q_j^*, c_i)$$

and for each $\theta \in \Omega$,

$$E^*(\theta) = \arg\max_{e \in \{A,B,\phi\}} u_E(e, Q_A^*, Q_B^*, \theta),$$

where u_i and u_E are the expected payoff of the worker from group i and the payoff of the firm, respectively.

[10] The main difficulty with the general case is that we cannot find an explicit relation between k_i^* and k_j^* in the relationships in (5).

Proof. Part 1. Let $k_i = k_j = k$. Then, $\Phi(k,k) = \beta(k,k)$. From (11),

$$\beta(k,k) = \int_{s(k_i)}^{\overline{\theta}} [F(k)G_1(\theta_i) + (1 - F(k))G_0(\theta_i)]\,dG_1(\theta_i) - \int_{s(k_i)}^{\overline{\theta}} [F(k)G_1(\theta_i) + (1 - F(k))G_0(\theta_i)]\,dG_0(\theta_i). \tag{6}$$

$[F(k)G_1(\theta_i) + (1 - F(k))G_0(\theta_i)]$ is a strictly increasing function of θ_i, and its first derivative exists and is continuous. In addition, it is bounded. Since $G_1(\theta_i) < G_0(\theta_i)$ for all θ_i, by Theorem 1 in Hadar & Russell (1971), $\beta(k,k) > 0$ for all $k > \underline{k}$. Let v be sufficiently large such that there exists $k' \in (\underline{c}, \overline{c})$ such that $\beta(k',k')v > k'$. By Lemma 1 (i) and (iv), $\beta(\underline{k},\underline{k}) = 0$, so $\beta(\underline{k},\underline{k})v < \underline{k} \in (\underline{c},\overline{c})$, and $\beta(k,k)$ is a continuous function of k on $[\underline{c},\overline{c})$, the intermediate value theorem implies that there exists $k_s^* \in (\underline{k}, k')$ such that $\beta(k_s^*, k_s^*)v = k_s^*$. By Lemma 1 (ii), $\beta(\overline{c},\overline{c}) = 0$, so $\beta(\overline{c},\overline{c})\overline{c} < \overline{c}$, and $\beta(k,k)$ is a continuous function of k on $[\underline{c},\overline{c})$. Similarly, there exists $k_s^{**} \in (k', \overline{c})$ such that $\beta(k_s^{**}, k_s^{**})v = k_s^{**}$.

Part 2. Let $k_j = \underline{c}$. From (8),

$$\beta(k,\underline{c}) = G_0(s(k)) - G_1(s(k)).$$

Hence, $\Phi(k,\underline{c}) = 1/2\beta(k,k) + 1/2\beta(k,\underline{c}) > 0$ for all $k > \underline{k}$. Let v be large enough that there exists $k' \in (\underline{c},\overline{c})$ such that $\Phi(k',\underline{c})v > k'$. Using the similar procedure above, now with Lemma 1 (iii), there exist $k_e^*, k_e^{**} \in (\underline{k},\overline{c})$ such that $\Phi(k_e^*,\underline{c})v = k_e^*$ and $\Phi(k_e^{**},\underline{c})v = k_e^{**}$. By Lemma 1 (i), $\Phi(\underline{c},k) = 0$ for all k, so $\Phi(\underline{c},k)v \le \underline{c}$. □

We can find a relationship between non-trivial symmetric equilibria and exclusive equilibria. If the advantaged group has two cutoffs $k_e^{**} > k_e^* > \underline{c}$ in exclusive equilibria, then there exist non-trivial multiple symmetric equilibria such that $k_s^* < k_e^*$ and $k_s^{**} > k_e^{**}$.

Proposition 2. *If there exist multiple exclusive asymmetric equilibria such that $(k_e^*, \underline{c})$ and $(k_e^{**}, \underline{c})$ with $k_e^{**} > k_e^*$, then there exist non-trivial multiple symmetric equilibria such that (k_s^*, k_s^*) and (k_s^{**}, k_s^{**}) with $k_s^* < k_e^*$ and $k_s^{**} > k_e^{**}$.*

Proof. For each k, let $\Phi(k,k) - \Phi(k,\underline{c}) = 1/2[\beta(k,k) - \beta(k,\underline{c})]$.

$$\beta(k,k) - \beta(k,\underline{c}) = \int_{[s(k),\overline{\theta}]} [F(k)G_1(\theta_i) + (1 - F(k))G_0(\theta_i) - 1]\,dG_1(\theta_i) - \int_{[s(k),\overline{\theta}]} [F(k)G_1(\theta_i) + (1 - F(k))G_0(\theta_i) - 1]\,dG_0(\theta_i).$$

$[F(k)G_1(\theta_i)+(1-F(k))G_0(\theta_i)-1]$ is a strictly increasing function of θ_i, and its first derivative exists and is continuous. In addition, it is bounded. Since $G_1(\theta_i)<G_0(\theta_i)$ for all θ_i, by Theorem 1 in Hadar & Russell (1971), $\beta(k,k)-\beta(k,\underline{c})>0$ for all $k>\underline{k}$. Let $k_e^{**}>k_e^*$. Given $\Phi(k_e^*,\underline{c})v=k_e^*$, $\Phi(k_e^*,k_e^*)v>\Phi(k_e^*,\underline{c})v=k_e^*$, so there exists $k_s^*\in(\underline{k},k_e^*)$ such that $\beta(k_s^*,k_s^*)v=k_s^*$. On the other hand, given $\Phi(k_e^{**},\underline{c})v=k_e^{**}$, $\Phi(k_e^{**},k_e^{**})v>\Phi(k_e^{**},\underline{c})v=k_e^{**}$, so there exists $k_s^{**}\in(k_e^{**},\overline{c})$ such that $\beta(k_s^{**},k_s^{**})v=k_s^{**}$. □

It is of interest to note that given exclusive equilibria, there can be a "good" symmetric equilibrium in which both groups' cutoffs are higher than that for the advantaged group in any exclusive equilibrium. However, the movement from an exclusive equilibrium to the good symmetric equilibrium may not be a Pareto improvement since a more qualified disadvantaged group can negatively affect them through competition.

Proposition 3. *Consider a movement from an exclusive equilibrium $(k_e^*,\underline{c})$ to a symmetric equilibrium (k_s^*,k_s^*) with $k_s^*>k_e^*$. If $s(k_e^*)-s(k_s^*)>0$ is sufficiently small, then each worker type c_i in the advantaged group is worse off.*

Proof. For each q, denote

$$\Phi(q,k_i,k_j)=1/2P(q_i,k_i,k_i)+1/2P(q_i,k_i,k_j).$$

Consider the changes in the probability P:

$$\begin{aligned}&\Phi(q,k_e^*,\underline{c})-\Phi(q,k_s^*,k_s^*)\\&=1/2P(q,k_e^*,k_e^*)+1/2P(q,k_e^*,\underline{c})-[1/2P(q,k_s^*,k_s^*)+1/2P(q,k_s^*,k_s^*)]\\&=1/2[P(q,k_e^*,k_e^*)-P(q,k_s^*,k_s^*)]+1/2[P(q,k_e^*,\underline{c})-P(q,k_s^*,k_s^*)].\end{aligned}$$

From the first term,

$$\begin{aligned}&P(q,k_e^*,k_e^*)-P(q,k_s^*,k_s^*)=\int_{s(k_e^*)}^{\overline{\theta}}[F(k_e^*)G_1(\theta_i)+(1-F(k_e^*))G_0(\theta_i)]dG_q(\theta_i)\\&-\int_{s(k_s^*)}^{\overline{\theta}}[F(k_s^*)G_1(\theta_i)+(1-F(k_s^*))G_0(\theta_i)]dG_q(\theta_i)\\&=\int_{s(k_s^*)}^{\overline{\theta}}[F(k_s^*)-F(k_e^*)][G_0(\theta_i)-G_1(\theta_i)]dG_q(\theta_i)\\&-\int_{s(k_s^*)}^{s(k_e^*)}[F(k_e^*)G_1(\theta_i)+(1-F(k_e^*))G_0(\theta_i)]dG_q(\theta_i).\end{aligned}$$

From the second term,

$$\begin{aligned}
&P(q,k_e^*,\underline{c}) - P(q,k_s^*,k_s^*)\\
&= \int_{s(k_e^*)}^{\overline{\theta}} dG_q(\theta_i) - \int_{s(k_s^*)}^{\overline{\theta}} [F(k_s^*)G_1(\theta_i) + (1-F(k_s^*))G_0(\theta_i)]dG_q(\theta_i)\\
&= \int_{s(k_s^*)}^{\overline{\theta}} \{F(k_s^*)[1-G_1(\theta_i)] + (1-F(k_s^*))[1-G_0(\theta_i)]\}dG_q(\theta_i) - \int_{s(k_s^*)}^{s(k_e^*)} dG_q(\theta_i).
\end{aligned}$$

If $s(k_e^*) - s(k_s^*) > 0$ is sufficiently small, for each q, $\Phi(q,k_e^*,\underline{c}) - \Phi(q,k_s^*,k_s^*) > 0$. For $c_i \in [\underline{c},k_e^*]$, $\Phi(1,k_e^*,\underline{c})v - c_i > \Phi(1,k_s^*,k_s^*)v - c_i$. For $c_i \in (k_e^*,k_s^*)$, $\Phi(0,k_e^*,\underline{c})v > \Phi(1,k_e^*,\underline{c})v - c_i > \Phi(1,k_s^*,k_s^*)v - c_i$. For $c_i \in [k_s^*,\overline{c}]$, $\Phi(0,k_e^*,\underline{c})v > \Phi(0,k_s^*,k_s^*)v$. □

The positive effect from the decrease in the test standard is not sufficiently large compared with the negative effect from greater competition.

3. CONCLUDING REMARKS

We analyze a statistical discrimination model featuring competition between groups. Pareto optimality may not suffice as a welfare criterion by which to judge discrimination. A stronger welfare criterion for measuring group inequality is needed, which would also make it possible to assess the welfare implications of affirmative action policies under this type of environment.

The model can be extended into two directions: two groups have two different cost distributions, and the firm's return is also affected by each worker's human capital investment costs. We leave them for future research.

APPENDIX:

Proof. [Proof of Lemma 1]Denote by $\mathscr{B}(k_i,k_j)$ the set of the test results for which the employer chooses a worker from group i:

$$\mathscr{B}(k_i,k_j) \equiv \left\{\theta \in \Theta(k_i) \mid \mu(\theta_i,k_i) > \mu(\theta_j,k_j)\right\}. \tag{7}$$

(i) For each $k_i \in [\underline{c},\underline{k}]$, (3) implies that for each q_i, $P(q_i,\underline{c},k_j) = 0$ for all $k_j \in [\underline{c},\overline{c}]$. (ii) (3) implies $\Theta(\overline{c}) = \Theta$, and $\mu(\theta_i,\overline{c}) = 1$, so $\mathscr{B}(\overline{c},k_j) = \Theta$ for all $k_j \in [\underline{c},\overline{c})$. For each q_i, $P(q_i,\overline{c},k_j) = 1$ for all $k_j \in [\underline{c},\overline{c}]$. In addition, for each q_i,

$P(q_i,\overline{c},\overline{c})=\frac{1}{2}$. (iii) Case 1: $k_j=\underline{c}$. $\mu(\theta_j,\underline{c})=0$, so $\mathscr{B}(k_i,\underline{c})=\Theta(k_i)$ for all $k_i\in(\underline{c},\overline{c}]$. From (4), for each q_i,

$$P(q_i,k_i,\underline{c})=\int_{s(k_i)}^{\overline{\theta}}dG_{q_i}(\theta_i)=1-G_{q_i}(s(k_i)). \tag{8}$$

Then, $P(q_i,k_i,\underline{c})$ is a continuous function of k_i on $(\underline{c},\overline{c}]$. In addition, by Lemma 1 (i), $\lim_{k_i\to\underline{c}}P(q_i,k_i,\underline{c})=0=P(q_i,\underline{c},\underline{c})$. Case 2: $k_j\in(\underline{c},\overline{c})$. Since ϕ is strictly decreasing, for $k_i\in(\underline{c},\overline{c})$, (7) can be written as

$$\mathscr{B}(k_i,k_j)=\left\{\theta\in\Theta(k_i)\mid\phi^{-1}\left(\phi(\theta_i)\frac{\pi(k_i)}{\pi(k_j)}\right)>\theta_j\right\}.$$

Form (4), for each q_i,

$$P(q_i,k_i,k_j)=F(k_j)\int_{\mathscr{B}(k_i,k_j)}dG_{q_i}(\theta_i)\times G_1(\theta_j)+(1-F(k_j))\int_{\mathscr{B}(k_i,k_j)}dG_{q_i}(\theta_i)\times G_0(\theta_j), \tag{9}$$

where

$$\int_{\mathscr{B}(k_i,k_j)}dG_{q_i}(\theta_i)\times G_{q_j}(\theta_j)=\int_{s(k_i)}^{\overline{\theta}}G_{q_j}\left(\phi^{-1}\left(\phi(\theta_i)\frac{\pi(k_i)}{\pi(k_j)}\right)\right)dG_{q_i}(\theta_i). \tag{10}$$

Since F, (G_1,G_0) and ϕ are continuous, $P(q_i,k_i,k_j)$ is a continuous function of k_i on $(\underline{c},\overline{c})$. In addition, by Lemma 1 (i) and (ii), for each q_i, $\lim_{k_i\to\underline{c}}P(q_i,k_i,k_j)=0=P(q_i,\underline{c},k_j)$ and $\lim_{k_i\to\overline{c}}P(q_i,k_i,k_j)=1=P(q_i,\overline{c},k_j)$. (iv) Let $k_i=k_j=k\in(\underline{c},\overline{c})$. Form (9), for each q_i,

$$P(q_i,k,k)=F(k)\int_{s(k)}^{\overline{\theta}}G_1(\theta_i)\,dG_{q_i}(\theta_i)+(1-F(k))\int_{s(k)}^{\overline{\theta}}G_0(\theta_i)\,dG_{q_i}(\theta_i). \tag{11}$$

Since F, (G_1,G_0) and ϕ are continuous, $P(q_i,k,k)$ is a continuous function of k on $(\underline{c},\overline{c})$. In addition, by Lemma 1 (i), for each q_i, $\lim_{k\to\underline{c}}P(q_i,k,k)=0=P(q_i,\underline{c},\underline{c})$ □

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